CHAPTER 6 Natural Convection Systems

6.1 Physical Mechanism Of Natural Convection

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipment such as power transistors, TVs, and VCRs; heat transfer from electric baseboard heaters or steam radiators; heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings. Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces. We know that a hot boiled egg (or a hot baked potato) on a plate eventually cools to the surrounding air temperature (Fig. 6–1). The egg is cooled by transferring heat by convection to the air and by radiation to the surrounding surfaces. Disregarding heat transfer by radiation, the physical mechanism of cooling a hot egg (or any hot object) in a cooler environment can be explained as follows:

As soon as the hot egg is exposed to cooler air, the temperature of the outer surface of the egg shell will drop somewhat, and the temperature of the air adjacent to the shell will rise as a result of heat conduction from the shell to the air. Consequently, the egg will soon be surrounded by a thin layer of warmer air, and heat will then be transferred from this warmer layer to the outer layers of air. The cooling process in this case would be rather slow since the egg would always be blanketed by warm air, and it would have no direct contact with the cooler air farther away. We may not notice any air motion in the vicinity of the egg, but careful measurements indicate otherwise.

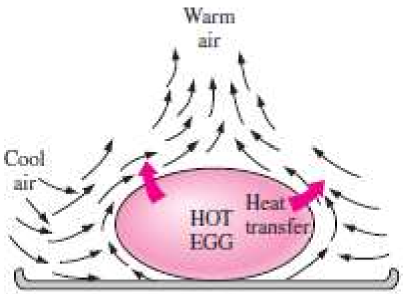
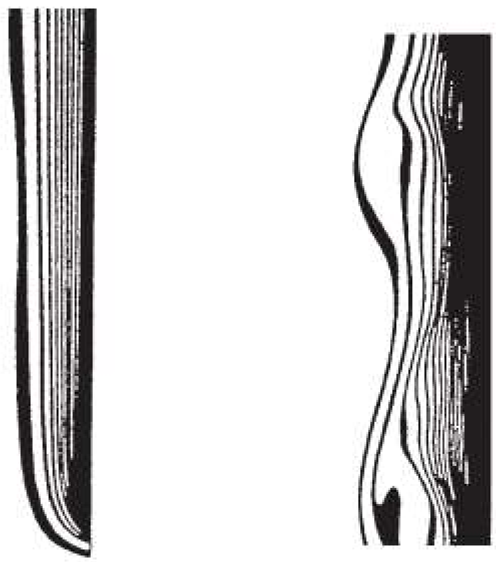


Figure 6–1 The cooling of a boiled egg in a cooler environment by natural convection.

Most heat transfer correlations in natural convection are based on experimental measurements. The lines of *constant temperature* for a natural convection on a hot vertical flat plate are shown in the Figure 6-2. The smooth and parallel lines in (*a*) indicate that the flow is *laminar,* whereas the eddies and irregularities in (*b*) indicate that the flow is *turbulent.* Note that the lines are closest near the surface, indicating a *higher temperature gradient.*



(*a*) Laminar flow (*b*) Turbulent flow

Isotherms in natural convection over a hot plate in air.

figure 6-2: a. Free convection laminar flow over isothermal hot plate, b. Free convection turbulent flow over isothermal hot plate.

Natural Convection Over Surfaces

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermo-physical properties of the fluid involved. The velocity and temperature profiles for natural convection over a vertical hot plate are shown in the Figure 6-3. Note that as in forced convection, the thickness of the boundary layer increases in the flow direction. Unlike forced convection, however, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate. This is expected since the fluid beyond the boundary layer is motionless. Thus, the fluid velocity increases with distance from the surface, reaches a maximum, and gradually decreases to zero at a distance sufficiently far from the surface.

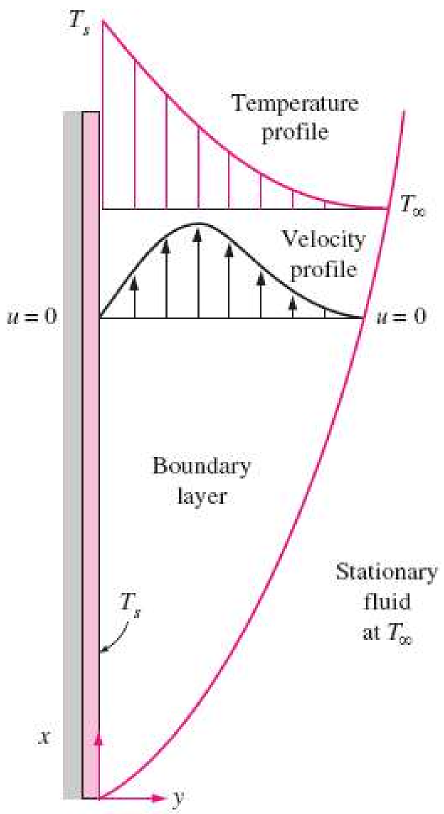


Figure 6-3: The velocity and temperature profiles for natural convection over a vertical hot plate.

With the exception of some simple cases, heat transfer relations in natural convection are based on experimental studies. The simple empirical correlations for the average *Nusselt number* Nu in natural convection are of the form,

$\overline{Nu}_f = C\left(Gr_f Pr_f\right)^m$ 6.1

Where C and m is the constants obtained from table 6-1.

The product of the Grashof and Prandtl numbers is called the Rayleigh number: $Ra = Gr\ Pr$

$Gr = \frac{g\beta(T_W - T_\infty)L_c^3}{v^2}$

Where:

Gr : Grashof number is the dimensionless used in the free convection.

g : gravitational acceleration, =9.81 m/s^2

$\beta = \frac{1}{T_f}$, T_f in kelvin.

The turbulent case $Gr_f Pr_f > 10^9$

Characteristic Dimensions

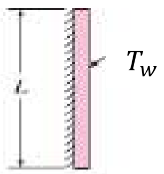
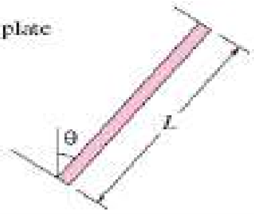
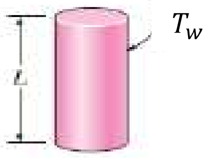
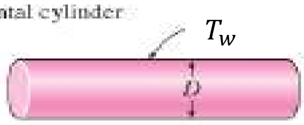
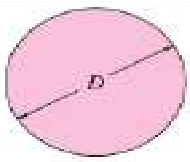
Geometry	Characteristic length L_c
Vertical plate 	L
Inclined plate 	L
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/P
Vertical cylinder 	L
Horizontal cylinder 	D
Sphere 	D

TABLE 6–1 Constants for use with Euation (6.1) for isothermal surfaces.

Geometry	$Gr_f\ Pr_f$	C	m
Vertical planes and cylinders	10^{-1}–10^{4}	Use Fig. 7-5	Use Fig. 7-5
	10^{4}–10^{9}	0.59	$\frac{1}{4}$
	10^{9}–10^{13}	0.021	$\frac{2}{5}$
	10^{9}–10^{13}	0.10	$\frac{1}{3}$
Horizontal cylinders	0–10^{-5}	0.4	0
	10^{-5}–10^{4}	Use Fig. 7-6	Use Fig. 7-6
	10^{4}–10^{9}	0.53	$\frac{1}{4}$
	10^{9}–10^{12}	0.13	$\frac{1}{3}$
	10^{-10}–10^{-2}	0.675	0.058
	10^{-2}–10^{2}	1.02	0.148
	10^{2}–10^{4}	0.850	0.188
	10^{4}–10^{7}	0.480	$\frac{1}{4}$
	10^{7}–10^{12}	0.125	$\frac{1}{3}$
Upper surface of heated plates or lower surface of cooled plates	2×10^{4}–8×10^{6}	0.54	$\frac{1}{4}$
Upper surface of heated plates or lower surface of cooled plates	8×10^{6}–10^{11}	0.15	$\frac{1}{3}$
Lower surface of heated plates or upper surface of cooled plates	10^{5}–10^{11}	0.27	$\frac{1}{4}$
Vertical cylinder, height = diameter characteristic length = diameter	10^{4}–10^{6}	0.775	0.21
Irregular solids, characteristic length = distance fluid particle travels in boundary layer	10^{4}–10^{9}	0.52	$\frac{1}{4}$

Constant Heat Flux Surfaces

$$Gr_x^* = Gr_x Nu_x = \frac{g\beta q_w x^4}{k v^2}$$

6.2

Where

$q_w = \frac{q}{A_s}$

A_s : Surface area, m^2.

$Gr_x^* Pr > 2 * 10^{13}$ turbulent flow

The *local* heat-transfer coefficients were correlated by the following relation for the laminar range:

$Nu_{xf} = \frac{hx}{k_f} = 0.6\left(Gr_x^* Pr_f\right)^{\frac{1}{5}}$ $10^5 < Gr_x^* Pr_f < 10^{11}$ laminar flow 6.3

Thus, for the laminar region, using Equation (6.3) to evaluate h_x,

$\bar{h} = \frac{5}{4} h_{x=L}$

For the turbulent region, the local heat-transfer coefficients are correlated with

$Nu_{xf} = \frac{hx}{k_f} = 0.17\left(Gr_x^* Pr_f\right)^{\frac{1}{4}}$ $2 * 10^{13} < Gr_x^* Pr_f < 10^{16}$ turbulent 6.4

Thus, for the turbulent region, using Equation (6.4) to evaluate h_x,

$\bar{h} = h_{x=L}$

All properties in Equations (6.3) and (6.4) are evaluated at the local film temperature.

$T_f = \frac{T_w + T_\infty}{2}$

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid surface at a uniform temperature T_w to the surrounding fluid is expressed by Newton's law of cooling as

$q = hA(T_w - T_\infty)$

Example 6.1: Constant Heat Flux from Vertical Plate

In a plant location near a furnace, a net radiant energy flux of 800 W/m^2 is incident on a vertical metal surface 3.5 m high and 2 m wide. The metal is insulated on the back side and painted black so that all the incoming radiation is lost by free convection to the surrounding air at 30°C. What average temperature will be attained by the plate?

Solution

We treat this problem as one with constant heat flux on the surface. Since we do not know the surface temperature, we must make an estimate for determining T_f and the air properties. An *approximate* value of h for free-convection problems is 10 W/m^2 · °C, and so, *approximately*,

$$\Delta T = \frac{q_w}{h} \approx \frac{800}{10} = 80^\circ\mathrm{C}$$

Then

$$T_f \approx \frac{80}{2} + 30 = 70^\circ\mathrm{C} = 343\ \mathrm{K}$$

At 70°C the properties of air are

$$\nu = 2.043 \times 10^{-5}\ \mathrm{m^2/s} \qquad \beta = \frac{1}{T_f} = 2.92 \times 10^{-3}\ \mathrm{K^{-1}}$$

$$k = 0.0295\ \mathrm{W/m \cdot ^\circ C} \qquad \mathrm{Pr} = 0.7$$

From Equation (6.2), with .5 m,

$$\mathrm{Gr}_x^* = \frac{g\beta q_w x^4}{k\nu^2} = \frac{(9.8)(2.92 \times 10^{-3})(800)(3.5)^4}{(0.0295)(2.043 \times 10^{-5})^2} = 2.79 \times 10^{14}$$

We may t Use Equation (6.4), to evaluate h_x:

$$h_x = \frac{k}{x}(0.17)(\mathrm{Gr}_x^*\,\mathrm{Pr})^{1/4}$$

$$= \frac{0.0295}{3.5}(0.17)(2.79 \times 10^{14} \times 0.7)^{1/4}$$

$$= 5.36\ \mathrm{W/m^2 \cdot ^\circ C}\ [0.944\ \mathrm{Btu/h \cdot ft^2 \cdot ^\circ F}]$$

$\bar{h} = h_{x=L} = 5.36$ for turbulent flow with constant heat flux.

$$\Delta T = \frac{q_w}{h} = \frac{800}{5.36} = 149^\circ\mathrm{C}$$

Our new film temperature would be

$$T_f = 30 + \frac{149}{2} = 104.5^\circ\mathrm{C}$$

At 104.5°C the properties of air are

$$\nu = 2.354 \times 10^{-5}\ \mathrm{m^2/s} \qquad \beta = \frac{1}{T_f} = 2.65 \times 10^{-3}/\mathrm{K}$$

$$k = 0.0320\ \mathrm{W/m \cdot ^\circ C} \qquad \mathrm{Pr} = 0.695$$

Then

$$Gr_x^* = \frac{(9.8)(2.65 \times 10^{-3})(800)(3.5)^4}{(0.0320)(2.354 \times 10^{-5})^2} = 1.75 \times 10^{14}$$

and h_x is calculated from

$$h_x = \frac{k}{x}(0.17)(Gr_x^* Pr)^{1/4}$$
$$= \frac{(0.0320)(0.17)}{3.5}[(1.758 \times 10^{14})(0.695)]^{1/4}$$
$$= 5.17\ W/m^2 \cdot {}^\circ C\ [-0.91\ Btu/h \cdot ft^2 \cdot {}^\circ F]$$

Our new temperature difference is calculated as

$$\Delta T = (T_w - T_\infty)_{av} = \frac{q_w}{h} = \frac{800}{5.17} = 155^\circ C$$

The average wall temperature is therefore

$$T_{w,av} = 155 + 30 = 185^\circ C$$

Another iteration on the value of T_f is not warranted by the improved accuracy that would result.

Example 6.2: Heat Transfer from Horizontal Tube in Water

A 2.0-cm-diameter horizontal heater is maintained at a surface temperature of 38°C and submerged in water at 27°C. Calculate the free-convection heat loss per unit length of the heater.

Solution

The film temperature is

$$T_f = \frac{38 + 27}{2} = 32.5^\circ C$$

From Appendix A the properties of water are

$$k = 0.630\ W/m \cdot {}^\circ C$$

and the following term is particularly useful in obtaining the Gr Pr product when it is multiplied by $d^3 \Delta T$:

$$\frac{g\beta\rho^2 c_p}{\mu k} = 2.48 \times 10^{10} \qquad [1/m^3 \cdot {}^\circ C]$$

$$\mathrm{Gr\,Pr} = (2.48 \times 10^{10})(38 - 27)(0.02)^3 = 2.18 \times 10^6$$

From Table 6-1, we get $C = 0.53$ and $m = \frac{1}{4}$, so that

$$\mathrm{Nu} = (0.53)(2.18 \times 10^6)^{1/4} = 20.36$$

$$h = \frac{(20.36)(0.63)}{0.02} = 642\ \mathrm{W/m^2 \cdot {}^\circ C}$$

The heat transfer is thus

$$\frac{q}{L} = h\pi d(T_w - T_\infty)$$
$$= (642)\pi(0.02)(38 - 27) = 443\ \mathrm{W/m}$$

Heat Transfer from Fine Wire in Air

Example 6.3:

A fine wire having a diameter of 0.02 mm is maintained at a constant temperature of 54°C by an electric current. The wire is exposed to air at 1 atm and 0°C. Calculate the electric power necessary to maintain the wire temperature if the length is 50 cm.

Solution

The film temperature is $T_f = (54 + 0)/2 = 27°\mathrm{C} = 300\ \mathrm{K}$, so the properties are

$$\beta = 1/300 = 0.00333 \qquad \nu = 15.69 \times 10^{-6}\ \mathrm{m^2/s}$$
$$k = 0.02624\ \mathrm{W/m \cdot {}^\circ C} \qquad \mathrm{Pr} = 0.708$$

$$Ra = GrPr = \frac{g\beta(T_w - T_\infty)L_c^3}{\nu^2} Pr$$

The Gr Pr product is then calculated as

$$\mathrm{Gr\,Pr} = \frac{(9.8)(0.00333)(54 - 0)(0.02 \times 10^{-3})^3}{(15.69 \times 10^{-6})^2}(0.708) = 4.05 \times 10^{-5}$$

From Table 6-1, we find $C = 0.675$ and $m = 0.058$ so that

$$\overline{\mathrm{Nu}} = (0.675)(4.05 \times 10^{-5})^{0.058} = 0.375$$

and

$$\bar{h} = \overline{\mathrm{Nu}}\left(\frac{k}{d}\right) = \frac{(0.375)(0.02624)}{0.02 \times 10^{-3}} = 492.6\ \mathrm{W/m^2 \cdot {}^\circ C}$$

The heat transfer or power required is then

$$q = \bar{h}A\,(T_w - T_\infty) = (492.6)\pi(0.02 \times 10^{-3})(0.5)(54 - 0) = 0.836\ \mathrm{W}$$

Heated Horizontal Pipe in Air — Example 6.4:

A horizontal pipe 1 ft (0.3048 m) in diameter is maintained at a temperature of 250°C in a room where the ambient air is at 15°C. Calculate the free-convection heat loss per meter of length.

Solution

$$T_f = \frac{T_w + T_\infty}{2} = \frac{250 + 15}{2} = 132.5°\text{C} = 405.5\ \text{K}$$

$$k = 0.03406\ \text{W/m}\cdot°\text{C} \qquad \beta = \frac{1}{T_f} = \frac{1}{405.5} = 2.47 \times 10^{-3}\ \text{K}^{-1}$$

$$\nu = 26.54 \times 10^{-6}\ \text{m}^2/\text{s} \qquad \text{Pr} = 0.687$$

$$\begin{aligned}\text{Gr}_d\,\text{Pr} &= \frac{g\beta(T_w - T_\infty)d^3}{\nu^2}\text{Pr} \\ &= \frac{(9.8)(2.47 \times 10^{-3})(250 - 15)(0.3048)^3(0.687)}{(26.54 \times 10^{-6})^2} \\ &= 1.571 \times 10^8\end{aligned}$$

From Table 6-1, $C = 0.53$ and $m = \frac{1}{4}$, so that

$$\text{Nu}_d = 0.53(\text{Gr}_d\,\text{Pr})^{1/4} = (0.53)(1.571 \times 10^8)^{1/4} = 59.4$$

$$h = \frac{k\text{Nu}_d}{d} = \frac{(0.03406)(59.4)}{0.3048} = 6.63\ \text{W/m}^2\cdot°\text{C} \quad [1.175\ \text{Btu/h}\cdot\text{ft}^2\cdot°\text{F}]$$

The heat transfer per unit length is then calculated from

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = 6.63\pi(0.3048)(250 - 15) = 1.49\ \text{kW/m} \quad [1560\ \text{Btu/h}\cdot\text{ft}]$$

Example 6.5: Cube Cooling in Air

A cube, 20 cm on a side, is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer.

Solution

This is an irregular solid so we use the information in the last entry of Table 7-1 in the absence of a specific correlation for this geometry. The properties were evaluated in Example 7-2 as

$$\beta = 3.25 \times 10^{-3} \qquad k = 0.02685$$
$$\nu = 17.47 \times 10^{-6} \qquad \mathrm{Pr} = 0.7$$

The characteristic length is the distance a particle travels in the boundary layer, which is $L/2$ along the bottom plus L along the side plus $L/2$ on the top, or $2L = 40$ cm. The Gr Pr product is thus:

$$\mathrm{Gr\,Pr} = \frac{(9.8)(3.25 \times 10^{-3})(60-10)(0.4)^3}{(17.47 \times 10^{-6})^2}(0.7) = 2.34 \times 10^8$$

From the last entry in **Table 6-1** we find $C = 0.52$ and $n = 1/4$ and calculate the Nusselt number as

$$\mathrm{Nu} = (0.52)(2.34 \times 10^8)^{1/4} = 64.3$$

and

$$\bar{h} = \mathrm{Nu}\frac{k}{L} = \frac{(64.3)(0.02685)}{(0.4)} = 4.32\ \mathrm{W/m^2 \cdot {}^\circ C}$$

The cube has six sides so the area is $6(0.2)^2 = 0.24\ \mathrm{m}^2$ and the heat transfer is

$$q = \bar{h}A(T_w - T_\infty) = (4.32)(0.24)(60-10) = 51.8\ \mathrm{W}$$